Position & Source: Position vector $\vec{r}$, source vector $\vec{r}'$, separation vector $\overrightarrow{\Delta r} = \vec{r} - \vec{r}'$

Fundamental Theorems of Vector Calculus:

$\int_{\vec{a}}^{\vec{b}} \nabla f \cdot \overrightarrow{dl} = f(\vec{b}) - f(\vec{a})$ $\quad$ $\int \nabla \cdot \vec{A}\, d\tau = \oint \vec{A} \cdot \overrightarrow{da}$ $\quad$ $\int (\nabla\times\vec{A}) \cdot \overrightarrow{da} = \oint \vec{A} \cdot \overrightarrow{dl}$

Cartesian Coordinates: $\overrightarrow{dl} = dx\hat{x} + dy\hat{y} + dz\hat{z}$ $\quad$ $d\tau = dx\, dy\, dz$

$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$ $\quad$ $\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$\nabla\times\vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z}$ $\quad$ $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

Spherical Coordinates: $x = r\, sin\theta\, cos\phi,\ y = r\, sin\theta\, sin\phi,\ z = r\, cos\theta$

$\overrightarrow{dl} = dr\hat{r} + r\, d\theta\hat{\theta} + r\, sin\theta\, d\phi\hat{\phi}$ $\quad$ $d\tau = r^2 sin\theta\, dr\, d\theta\, d\phi$

$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\, sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$ $\quad$ $\nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\, sin\theta}\frac{\partial(sin\theta\, A_\theta)}{\partial \theta} + \frac{1}{r\, sin\theta}\frac{\partial A_\phi}{\partial \phi}$

$\nabla\times\vec{A} = \frac{1}{r\, sin\theta}\left(\frac{\partial(sin\theta\, A_\phi)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial(r\, A_\phi)}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial(r\, A_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)\hat{\phi}$

$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$

Cylindrical Coordinates: $x = s\, cos\phi,\ y = s\, sin\phi,\ z = z$

$\overrightarrow{dl} = ds\hat{s} + s\, d\phi\hat{\phi} + dz\hat{z}$ $\quad$ $d\tau = s\, ds\, d\phi\, dz$

$\nabla f = \frac{\partial f}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial f}{\partial\phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$ $\quad$ $\nabla \cdot \vec{A} = \frac{1}{s}\frac{\partial(s\, A_s)}{\partial s} + \frac{1}{s}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$

$\nabla\times\vec{A} = \left(\frac{1}{s}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial(s\, A_\phi)}{\partial s} - \frac{\partial A_s}{\partial\phi}\right)\hat{z}$ $\quad$ $\nabla^2 f = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial f}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}$

Tensor Math: $(\vec{a} \cdot \overleftrightarrow{T})_j = \sum_i a_i T_{ij}$ $\quad$ $(\overleftrightarrow{T} \cdot \vec{a})_j = \sum_i T_{ji} a_i$

Lorentz Force: $\vec{F} = Q(\vec{E} + \vec{v}\times\vec{B})$, $\quad$ <u>On Wire:</u> $\vec{F}_{mag} = \int I(\overrightarrow{dl}\times\vec{B})$

Maxwell's Equations:

$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ $\quad$ $\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d}{dt}\int \vec{B} \cdot \overrightarrow{da}$

$\nabla \cdot \vec{E} = \rho/\varepsilon_0$ $\quad$ $\oint \vec{E} \cdot \overrightarrow{da} = Q_{enc}/\varepsilon_0$

$\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$ $\quad$ $\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I_{enc} + \mu_0\varepsilon_0\frac{d}{dt}\int \vec{E} \cdot \overrightarrow{da}$

$\nabla \cdot \vec{B} = 0$ $\quad$ $\oint \vec{B} \cdot \overrightarrow{da} = 0$

Fields in Matter: $\vec{P} = \vec{p}/volume$ $\quad \vec{D} = \varepsilon_0\vec{E} + \vec{P}$ $\quad \sigma_b = \vec{P}\cdot\hat{n}$ $\quad \rho_b = -\nabla\cdot\vec{P}$ $\quad J_p = \frac{\partial\vec{P}}{\partial t}$

$\vec{M} = \vec{m}/volume$ $\quad \vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$ $\quad \vec{K}_b = \vec{M}\times\hat{n}$ $\quad \vec{J}_b = \nabla\times\vec{M}$

$\nabla\cdot\vec{D} = \rho_f$ $\qquad \oint\vec{D}\cdot\overrightarrow{da} = Q_{f\_enc}$

$\nabla\times\vec{H} = \vec{J_f} + \frac{\partial\vec{D}}{\partial t}$ $\qquad \oint\vec{H}\cdot\overrightarrow{dl} = I_{f_{enc}} + \frac{d}{dt}\int\vec{D}\cdot\overrightarrow{da}$

Linear Materials: $\vec{P} = \varepsilon_0\chi_e\vec{E}$ $\qquad \vec{D} = \varepsilon\vec{E} = (1+\chi_e)\varepsilon_0\vec{E} = \varepsilon_r\varepsilon_0\vec{E}$

$\vec{M} = \chi_m\vec{H}$ $\qquad \vec{B} = \mu\vec{H} = (1+\chi_m)\mu_0\vec{H}$

Boundary Conditions: $\Delta D_\perp = \sigma_f$ $\qquad \Delta\vec{E}_{||} = 0$ $\qquad \Delta\vec{D}_{||} = \Delta\vec{P}_{||}$

$\Delta\vec{H}_{||} = \vec{K}_f\times\hat{n}$ $\qquad \Delta B_\perp = 0$ $\qquad \Delta H_\perp = -\Delta M_\perp$

Ohm's Law and EMF: $\vec{J} = \sigma\vec{E}$ $\qquad \mathcal{E} = \oint(\vec{F}/q)\cdot\overrightarrow{dl}$ $\qquad \mathcal{E}_{motional} = -\frac{d\Phi_B}{dt}$ $\qquad \frac{dW}{dt} = \mathcal{E}I$

Inductance: $M = \frac{\Phi_{B2}}{I_1} = \frac{\Phi_{B1}}{I_2}$ $\qquad L = \frac{\Phi_{B1}}{I_1}$ $\qquad \mathcal{E}_{induced} = -L\frac{dI}{dt}$

Continuity of Charge/Current: $\frac{\partial\rho}{\partial t} = -\nabla\cdot\vec{J}$ $\qquad \frac{\partial\rho_b}{\partial t} = -\nabla\cdot\overrightarrow{J_p}$

Energy & Momentum: $\vec{S} = \frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right)$ $\qquad u_{EM} = \frac{1}{2}\left(\varepsilon_0E^2 + \frac{B^2}{\mu_0}\right)$ $\qquad U_{EM} = \int u_{EM}\,d\tau$

$T_{ij} = \varepsilon_0\left(E_iE_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right)$ $\qquad \vec{g} = \varepsilon_0\left(\vec{E}\times\vec{B}\right) = \mu_0\varepsilon_0\vec{S}$ $\qquad \vec{p}_{EM} = \int\vec{g}\,d\tau$

$\frac{dW}{dt} = -\oint\vec{S}\cdot\overrightarrow{da} - \frac{dU_{EM}}{dt}$ $\qquad \frac{\partial u_{EM}}{\partial t} = -\nabla\cdot\vec{S}\ \ if\ \frac{dW}{dt} = 0$

$\vec{F} = \frac{d\vec{p}_{mech}}{dt} = \oint\overleftrightarrow{T}\cdot\overrightarrow{da} - \frac{d\vec{p}_{EM}}{dt}$ $\qquad \vec{f} = \nabla\cdot\overleftrightarrow{T} - \frac{\partial\vec{g}}{\partial t}$ $\qquad \frac{\partial\vec{g}}{\partial t} = \nabla\cdot\overleftrightarrow{T}\ \ if\ \vec{f} = 0$

<u>Linear materials:</u> $\vec{S} = \frac{1}{\mu}\left(\vec{E}\times\vec{B}\right)$ $\qquad u_{EM} = \frac{1}{2}\left(\varepsilon E^2 + \frac{B^2}{\mu}\right)$

EM Plane Waves:

Complex : $\vec{E}(\vec{r},t) = \widetilde{E_0}\exp\left(i(\vec{k}\cdot\vec{r} - \omega t)\right)\hat{n}$ $\qquad \vec{B}(\vec{r},t) = \frac{k}{\omega}\widetilde{E_0}\exp\left(i(\vec{k}\cdot\vec{r} - \omega t)\right)\left(\hat{k}\times\hat{n}\right)$

Real: $\vec{E}(\vec{r},t) = E_0\cos\left(\vec{k}\cdot\vec{r} - \omega t + \delta\right)\hat{n}$ $\qquad \vec{B}(\vec{r},t) = \frac{k}{\omega}E_0\cos\left(\vec{k}\cdot\vec{r} - \omega t + \delta\right)\left(\hat{k}\times\hat{n}\right)$

$\frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ $\qquad \langle u\rangle = \frac{1}{2}\varepsilon_0{E_0}^2$ $\qquad \langle\vec{S}\rangle = c\langle u\rangle\hat{k} = I\hat{k} = \frac{1}{2}c\varepsilon_0{E_0}^2\hat{k}$ $\qquad \langle\vec{g}\rangle = \frac{\langle\vec{S}\rangle}{c^2} = \frac{1}{2c}\varepsilon_0{E_0}^2\hat{k}$

<u>Linear materials:</u> $\frac{\omega}{k} = v = \frac{1}{\sqrt{\mu\varepsilon}}$ $\qquad \langle\vec{S}\rangle = v\langle u\rangle\hat{k} = I\hat{k} = \frac{1}{2}v\varepsilon{E_0}^2\hat{k}$ $\qquad n = \frac{c}{v} = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$

<u>Conductors</u>: $k_r = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{1 + (\sigma/\varepsilon\omega)^2} + 1\right]^{1/2}$ $\qquad k_i = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{1 + (\sigma/\varepsilon\omega)^2} - 1\right]^{1/2}$

$\tilde{k} = k_r + ik_i$ $\quad d = 1/k_i$ $\quad n = \frac{c}{v} = \frac{ck_r}{\omega}$ $\quad \frac{B_0}{E_0} = \frac{|k|}{\omega} = \frac{\sqrt{{k_r}^2 + {k_i}^2}}{\omega}$

<u>Reflection/Transmission:</u> $\theta_I = \theta_R$ $\qquad \frac{sin\theta_T}{sin\theta_I} = \frac{n_1}{n_2}$ $\quad \alpha = \frac{cos\theta_T}{cos\theta_I}$ $\quad \beta = \frac{\mu_1n_2}{\mu_2n_1} = \frac{\mu_1v_1}{\mu_2v_2}$

$\tilde{E}_{0R} = \frac{\alpha-\beta}{\alpha+\beta}\tilde{E}_{0I}$ $\quad \tilde{E}_{0T} = \frac{2}{\alpha+\beta}\tilde{E}_{0I}$ $\quad R = \langle S_R\rangle/\langle S_I\rangle$ $\quad T = \langle S_T\rangle cos\theta_T/\langle S_I\rangle cos\theta_I$ $\quad R+T=1$

Potentials: $\vec{B} = \nabla\times\vec{A}$ $\vec{E} = -\nabla V - \frac{\partial\vec{A}}{\partial t}$ Gauge Transformation: $\vec{A}' = \vec{A} + \nabla\lambda, V' = V - \frac{\partial\lambda}{\partial t}$

$\nabla^2 V + \frac{\partial}{\partial t}(\nabla\cdot\vec{A}) = -\frac{\rho}{\varepsilon_0}$ $(\nabla^2\vec{A} - \mu_0\varepsilon_0\frac{\partial^2\vec{A}}{\partial t^2}) - \nabla(\nabla\cdot\vec{A} + \mu_0\varepsilon_0\frac{\partial V}{\partial t}) = -\mu_0\vec{J}$

Coulomb Gauge: $\nabla\cdot\vec{A} = 0$ => $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$ $(\nabla^2\vec{A} - \mu_0\varepsilon_0\frac{\partial^2\vec{A}}{\partial t^2}) - \mu_0\varepsilon_0\nabla(\frac{\partial V}{\partial t}) = -\mu_0\vec{J}$

Lorenz Gauge: $\nabla\cdot\vec{A} = -\mu_0\varepsilon_0\frac{\partial V}{\partial t}$ => $\nabla^2 V - \mu_0\varepsilon_0\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon_0}$ $(\nabla^2\vec{A} - \mu_0\varepsilon_0\frac{\partial^2\vec{A}}{\partial t^2}) = -\mu_0\vec{J}$

Retarded Potentials: $t_r = t - \frac{\Delta r}{c}$ $V(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\vec{r}',t_r)}{\Delta r}d\tau'$ $\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}',t_r)}{\Delta r}d\tau'$

Liénard-Wiechert Potentials: $V(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\frac{qc}{(c\,\Delta r - \overrightarrow{\Delta r}\cdot\vec{v})}$ $\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\frac{qc\vec{v}}{(c\,\Delta r - \overrightarrow{\Delta r}\cdot\vec{v})} = \frac{\vec{v}}{c^2}V(\vec{r},t)$

$\vec{E}(\vec{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{\Delta r}{(\overrightarrow{\Delta r}\cdot\vec{u})^3}\left[(c^2 - v^2)\vec{u} + \overrightarrow{\Delta r}\times(\vec{u}\times\vec{a})\right]$ $\vec{B}(\vec{r},t) = \frac{1}{c}\widehat{\Delta r}\times\vec{E}$ $\vec{u} = c\widehat{\Delta r} - \vec{v}$

Constant Velocity: $\vec{E}(\vec{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{1 - v^2/c^2}{(1 - v^2 sin^2\theta/c^2)^{3/2}}\frac{\hat{R}}{R^2}$ $\vec{B}(\vec{r},t) = \frac{1}{c}\widehat{\Delta r}\times\vec{E} = \frac{1}{c^2}\vec{v}\times\vec{E}$

Radiation: Electric Dipole: $\vec{S}(\vec{r},t) \cong \frac{\mu_0}{c}\left\{\frac{p_0\omega^2}{4\pi}\frac{sin\theta}{r}\cos\left[\omega\left(t - \frac{r}{c}\right)\right]\right\}^2\hat{r}$ $P = \frac{\mu_0 {p_0}^2\omega^4}{12\pi c}$

Magnetic Dipole: $\vec{S}(\vec{r},t) \cong \frac{\mu_0}{c}\left\{\frac{m_0\omega^2}{4\pi c}\frac{sin\theta}{r}\cos\left[\omega\left(t - \frac{r}{c}\right)\right]\right\}^2\hat{r}$ $P = \frac{\mu_0 {m_0}^2\omega^4}{12\pi c^3}$

Multipole: $\vec{E}(\vec{r},t) \cong \frac{\mu_0}{4\pi r}[\hat{r}\times(\hat{r}\times\ddot{\vec{p}})]$ $\vec{B}(\vec{r},t) \cong -\frac{\mu_0}{4\pi rc}[\hat{r}\times\ddot{\vec{p}}]$ $\vec{S}(\vec{r},t) \cong \frac{\mu_0\ddot{p}^2}{16\pi^2 c}\frac{sin^2\theta}{r^2}\hat{r}$

Pt. Chrg: $\vec{E}_{rad}(\vec{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{\Delta r}{(\overrightarrow{\Delta r}\cdot\vec{u})^3}\left[\overrightarrow{\Delta r}\times(\vec{u}\times\vec{a})\right]$ $\vec{S}_{rad} = \frac{1}{\mu_0 c}{E_{rad}}^2\widehat{\Delta r}$ $P = \frac{\mu_0 q^2\gamma^6}{6\pi c}\left(a^2 - \left|\frac{\vec{v}\times\vec{a}}{c}\right|^2\right)$

Low Velocity: $\vec{S}_{rad} = \frac{\mu_0 q^2 a^2}{16\pi^2 c}\frac{sin^2\theta}{\Delta r^2}\widehat{\Delta r}$ $P = \oint\vec{S}\cdot\overrightarrow{da} = \frac{\mu_0 q^2 a^2}{6\pi c}$

Relativity: $x^\mu = (ct, x, y, z)$ $\eta^\mu = \frac{dx^\mu}{d\tau} = (c, v_x, v_y, v_z)/\sqrt{1 - v^2/c^2}$

$p^\mu = m\eta^\mu = (mc, mv_x, mv_y, mv_z)/\sqrt{1 - v^2/c^2} = (E/c, p_x, p_y, p_z)$

$K^\mu = \frac{dp^\mu}{d\tau} = (\frac{1}{c}\frac{dE}{dt}, \frac{dp_x}{dt}, \frac{dp_y}{dt}, \frac{dp_z}{dt})/\sqrt{1 - v^2/c^2} = (\frac{1}{c}\frac{dE}{dt}, F_x, F_y, F_z)/\sqrt{1 - v^2/c^2}$

$a^{\mu\prime} = (\gamma[a^0 - \beta a^1], \gamma[a^1 - \beta a^0], a^2, a^3)$ $\gamma = 1/\sqrt{1 - u^2/c^2}$ $\beta = u/c$

$a^\mu a_\mu = -a^0 a^0 + a^1 a^1 + a^2 a^2 + a^3 a^3 = Invariant\ under\ Lorentz\ transformations$

Relativistic Electrodynamics:

${E_x}' = E_x,\ {E_y}' = \gamma(E_y - uB_z),\ {E_z}' = \gamma(E_z + uB_y)$

${B_x}' = B_x,\ {B_y}' = \gamma\left(B_y + \frac{u}{c^2}E_z\right), {B_z}' = \gamma\left(B_z - \frac{u}{c^2}E_y\right)$

$F^{\mu\nu} = \left[\left[0, \frac{E_x}{c}, \frac{E_y}{c}, \frac{E_z}{c}\right], \left[-\frac{E_x}{c}, 0, B_z, -B_y\right], \left[-\frac{E_y}{c}, -B_z, 0, B_x\right], \left[-\frac{E_z}{c}, B_y, -B_x, 0\right]\right]$ $\frac{\partial F^{\mu\nu}}{\partial x^\nu} = \mu_0 J^\mu$

$G^{\mu\nu} = \left[\left[0, B_x, B_y, B_z\right], \left[-B_x, 0, -\frac{E_z}{c}, \frac{E_y}{c}\right], \left[-B_y, \frac{E_z}{c}, 0, -\frac{E_x}{c}\right], \left[-B_z, -\frac{E_y}{c}, \frac{E_x}{c}, 0\right]\right]$ $\frac{\partial G^{\mu\nu}}{\partial x^\nu} = 0$

$J^\mu = \rho_0\eta^\mu = (c\rho_0, \rho_0 v_x, \rho_0 v_y, \rho_0 v_z)/\sqrt{1 - v^2/c^2} = (c\rho, J_x, J_y, J_z)$ $\frac{\partial J^\mu}{\partial x^\mu} = 0$ $K^\mu = q\eta_\nu F^{\mu\nu}$

$A^\mu = (\frac{V}{c}, A_x, A_y, A_z)$ $F^{\mu\nu} = \frac{\partial A^\nu}{\partial x_\mu} - \frac{\partial A^\mu}{\partial x_\nu}$ $\frac{\partial}{\partial x_\nu}\frac{\partial}{\partial x^\nu}A^\mu = -\mu_0 J^\mu\ (Lorenz\ gauge)$